RELATIONS AND FUNCTIONS

Ordered Pair

Listing two members in a particular order, separated by a comma and enclosing the pair in parentheses is known as an ordered pair. In the ordered pair (a, b), a is called the first component or the first element or the first member or the first coordinate and b is called the second component or the second element or the second member or the second coordinate.

In an ordered pair (a, b), the order in which the elements a and b appear in the bracket is important. The ordered pair (1, 2) and (2, 1) though consist of the same numbers 1 and 2 are different as they represent different points in the co-ordinate plane.

Equality of two ordered pairs

Two ordered pairs (a_1, b_1) and (a_2, b_2) are said to be equal iff $a_1 = a_2$ and $b_1 = b_2$.

If ordered pairs (a_1, b_1) and (a_2, b_2) are equal, we write $(a_1,b_1) = (a_2, b_2)$ For examples -

$(1, 3) = (1, 3)$ but $(1, 3) \neq (3, 1)$ and $(1, 3) \neq (1, 2)$ etc.

Cartesian product of sets

The cartesian product of two sets A and B is the set of all those ordered pairs whose first co-ordinate is an element of A and the second co-ordinate is an element of B. This set is denoted by $A \times B$ and is read as 'A cross B' or 'product set of A and B'.

Symbolically, $A \times B = \{(a, b) : a \in A \text{ and } b \in B\}$.

Thus $(a, b) \in A \times B \Leftrightarrow a \in A$ and $b \in B$

Number of elements in the Cartesian Product

If A and B are two finite sets then $n(A \times B) = n(A).n(B)$ i.e. if A has m elements and B has n elements, then $A \times B$ has mn elements.

Note : The elements of $A \times B$ are also called ordered pairs or 2-tuples while the elements of $A \times B \times C$ are called ordered triplets or 3-tuples.

Illustration 1

If A = {1, 2, 3} and B = {3, 5} then find A × B and B × A also verify $A \times B \neq B \times A$.

$A \times B = \{1, 2, 3\} \times \{3, 5\}$

$= \{(1, 3), (1, 5), (2, 3), (2, 5), (3, 3), (3, 5)\}$

and $B \times A = \{3, 5\} \times \{1, 2, 3\}$

$= \{(3, 1), (3, 2), (3, 3), (5, 1), (5, 2), (5, 3)\}$

Clearly $A \times B \neq B \times A$

Thus, "cartesian product of sets is not commutative".

Illustration 2

If A = {1, 2} , B = {4, 5}, C = {6, 8} then find A × B × C.

$A \times B \times C = A \times [B \times C] = \{1, 2\} \times [\{4, 5\} \times \{6, 8\}]$

$= \{1, 2\} \times \{(4, 6), (4, 8), (5, 6), (5, 8)\}$

$= \{(1, 4, 6),(1, 4, 8),(1, 5, 6),(1, 5, 8),(2, 4, 6),(2, 4, 8),(2, 5, 6),(2, 5, 8)\}$

Illustration 3

Let A = {1, 2, 3, 4} and S = {(a, b) : a ∈ A, b ∈ A, a divides b} Write S explicitly

Since 1 divides 2, 3, 4 and 2 divides 4

3 does not divide any of 1, 2 and 4.

4 does not divide any of 1, 2 and 3

Also 1 divides 1, 2 divides 2, 3 divides 3 and 4 divides 4.

Hence $S = \{(a, b) : a \in A, b \in A, a \text{ divides } b\}$

$= \{(1, 2), (1, 3), (1, 4), (2, 4), (1, 1), (2, 2), (3, 3), (4, 4)\}$

Illustration 4

Let A = {1, 2} and B = {3, 4}. Write all subsets of A × B.

Given, A = {1, 2} and B = {3, 4}

∴ A × B= {(1, 3), (1, 4), (2, 3), (2, 4)}.

Subsets of A × B are

ϕ, {(1, 3)}, {(1, 4)}, {(2, 3)}, {(2, 4)}, {(1, 3), (1, 4)}, {(1, 3), (2, 3)}, {(1, 3), (2, 4)}, {(1, 4), (2, 3)}, {(1, 4), (2, 4)}, {(2, 3), (2, 4)}, {(1, 3), (1, 4), (2, 3)}, {(1, 3), (1, 4), (2, 4))}, {(1, 4), (2, 3), (2, 4)}, {(1, 3), (2, 3), (2, 4)}, A × B.

Illustration 5

If A and B are two sets given in such a way that A × B consists of 6 elements. And if three elements of A × B are (1, 3), (2, 5), (3, 5), then what are its remaining elements?

Since (1, 3) , (2, 5), (3, 5) ∈ A × B, so clearly 1, 2, 3 ∈ A and 3, 5 ∈ B.

Given, n(A × B) = 6 ⇒ n(A) . n(B) = 6.

But 1, 2, 3 ∈ A and 3, 5 ∈ B

Hence A = {1, 2, 3} and B = {3, 5}

∴ A × B = {(1, 3), (1, 5), (2, 3), (2, 5), (3, 3), (3, 5)}.

∴ Remaining elements of A × B are : (1, 5), (2, 3) and (3, 3)

Some Results on Cartesian Product of Sets

1. Let A, B, C be three sets. Then
   (a) A × (B ∪ C) = (A × B) ∪ (A × C)
   (b) A × (B ∩ C) = (A × B) ∩ (A × C)
   (c) A × (B – C) = (A × B) – (A × C)
   (d) (A – B) × C = (A × B) – (B × C)
   (e) (A ∩ B) × C = (A × C) ∩ (B × C)
   (f) (A ∪ B) × C = (A × C) ∪ (B × C)
2. If A, B and C be any sets and A ⊆ B, then A × C ⊆ B × C
3. If A ⊆ B and C ⊆ D then A × C ⊆ B × D
4. If A, B, C, D be any sets, then (A × B) ∩ (C × D) = (A ∩ C) ×(B ∩ D)
5. If A and B be any two non-empty sets, then
   A × B = B × A ⇔ A = B

Illustration 6

Prove that A ⊆ B and C ⊆ D ⇒ A × C ⊆ B × D.

Given A ⊆ B and C ⊆ D.

Let (x, y) be any element of A × C.

Then (x, y) ∈ A × C

⇒ x ∈ A, y ∈ C

⇒ x ∈ B, y ∈ D . [∵ A ⊆ B and C ⊆ D]

⇒ (x, y) ∈ B × D

Thus A × C ⊆ B × D.

Illustration 7

If A and B are any two non empty sets, then prove that $A \times B = B \times A \Leftrightarrow A = B$.

If A = B then we have to prove $A \times B = B \times A$

Now, $A \times B = A \times A$ $\quad [\because B = A]$

$= B \times A$ $\quad [\because A = B]$

If $A \times B = B \times A$ then we have to prove A = B

Let $a \in A$

Since B is non-empty $\therefore$ there exists $b \in B$

Now, $a \in A$ and $b \in B \Rightarrow (a, b) \in A \times B$

$\Rightarrow \quad (a, b) \in B \times A$

$\Rightarrow \quad a \in B$

Thus $a \in A \Rightarrow a \in B \therefore A \subseteq B$ ....(1)

Let $b \in B$

Since A is non-empty $\therefore$ there exist $a \in A$

Now , $b \in B$ and $a \in A \Rightarrow (b, a) \in B \times A$

$\Rightarrow (b, a) \in A \times B$

$\Rightarrow b \in A$

Thus $b \in B \Rightarrow A \therefore B \subseteq A$ ....(2)

From (1) and (2), we have A = B .

Illustration 8

Prove that $A \times \phi = \phi$ for any set A.

Let $(x, y) \in A \times \phi \Rightarrow y \in \phi$ (Contradiction)

$\therefore \quad A \times \phi = \phi$

Illustration 9

If A = {1, 2, 3}, B = {2, 3, 4}, C = {1, 3, 4} and D = {2, 4, 5}, then verify that $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$

$(A \times B)$ = {1, 2, 3} × {2, 3, 4}

= {(1, 2), (1, 3), (1, 4), (2, 2), (2, 3), (2, 4), (3, 2), (3, 3), (3, 4)}

$(C \times D)$ = {1, 3, 4} ({2, 4, 5}

= {(1, 2),(1, 4), (1, 5), (3, 2), (3, 4), (3, 5), (4, 2), (4, 4), (4, 5)}

$\therefore (A \times B) \cap (C \times D)$ = {(1, 2), (1, 4), (3, 2), (3, 4)}

Also, $(A \cap C)$ = {1, 3} and $(B \cap D)$ = {2, 4}. Therefore

$(A \cap C) \times (B \cap D)$ = {(1,2), (1, 4), (3, 2), (3, 4)}

Hence, $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$

Illustration 10

If A and B are two non empty sets having n elements in common, then prove that $A \times B$ and $B \times A$ have n^2 elements in common.

Let $C = A \cap B$. Then $C \times C = (A \cap B) \times (A \cap B) = (A \cap B) \times (B \cap A)$

$= (A \times B) \cap (B \times A)$ $\quad$ $[\because (A \cap C) \times (B \cap D) = (A \times B) \cap (C \times D)]$

Since $A \cap B$ has n elements, so C has n elements. Hence $C \times C$ has n^2 elements.

$\therefore$ $(A \times B) \cap (B \times A)$ has n^2 elements.

Hence $A \times B$ and $B \times A$ have n^2 elements in common.

Practice Assignment–I

1. Find the values of a and b in the following equal ordered pairs:
(i) $(a, 3) = (2, b)$ (ii) $(2a - 3, b + 2) = (3a + 1, 2b + 5)$

2. If $A = \{1, 3, 5\}$ and $B = \{2, 3\}$, then show that $A \times B \neq B \times A$

3. If $A = \{a, b\}$, $B = \{c, d\}$, $C = \{d, e\}$, then find
(i) $A \times (B \cup C)$ (ii) $A \times (B \cap C)$

4. If $A = \{1, 2\}$, $B = \{3, 4\}$, $C = \{4, 5\}$, find $A \times (B \cup C)$

5. If $A = \{1, 2, 3\}$, $B = \{4\}$, $C = \{5\}$, then verify that
(i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$
(ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$
(iii) $A \times (B - C) = (A \times B) - (A \times C)$

6. If $A = \{1, 4\}$, $B = \{2, 3, 6\}$ and $C = \{2, 3, 7\}$, then verify that:
(i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$
(ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

7. If $A = \{2, 3\}$, $B = \{6, 8\}$, $C = \{1, 2\}$ and $D = \{6, 9\}$ then verify that $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$

8. A and B are two sets given in such a way that $A \times B$ contains 6 elements. If three elements of $A \times B$ be (1, 3), (2, 5) and (3, 3), find its remaining elements.

9. Let $A = \{2, 4, 6, 8\}$ and $S = \{(a, b): a \in A, b \in A, a \text{ divides } b\}$. Write S explicitly.

10. Let A and B be two sets such that n(A) = 3 and n(B) = 2. If (x, 1), (y, 2), (z, 1) are in $A \times B$, find A & B, where x, y, z are distinct elements.

11. Let $A = \{1, 2\}$, $B = \{1, 2, 3, 4\}$, $C = \{5, 6\}$ and $D = \{5, 6, 7, 8\}$. Verify that $A \times C \subset B \times D$

12. The catresian product $A \times A$ has 9 elements and two of them are (–1, 0) and (0, 1). Find the set A and the remaining elements of $A \times A$.

13. If $A \times B = \{(2, 3), (2, 1), (3, 3), (3, 1), (4, 3), (4, 1)\}$, find A and B

14. A and B are two sets having 3 elements in common. If n(A) = 5, n(B) = 4, find $n(A \times B)$ and $n[(A \times B) \cap (B \times A)]$.

15. If $A = \{-1, 1\}$, find $A \times A \times A$.

Relations

A relation R, from a non-empty set A to another non-empty set B, is a subset of $A \times B$.

Thus, $R : A \to B \Leftrightarrow R \subseteq A \times B$

$\Leftrightarrow$ $\quad R \subseteq \{(a, b) : a \in A, b \in B)\}$

For examples :

(i) Let $A = \{1, 2, 4\}$ and $B = \{4, 6\}$
Let $R = \{(1, 4), (1, 6), (2, 4), (2, 6), (4, 6)\}$.
Here $R \subseteq A \times B$ and therefore, R is a relation from A to B.
Here relation R denotes the relation "is less than" $(1, 4) \in R$ means 1 is less than 4.

(ii) Let $A = \{1, 2\}$, $B = \{a, b, c\}$
Let $R = \{(1, a), (1, c)\}$. Here R is a subset of $A \times B$ and hence it is a relation from A to B.

(iii) Let $A = \{1, 2, 3\}$, $B = \{2, 3, 5, 7\}$. Let $R = \{(2, 3), (3, 5), (5, 7)\}$. Then R is not a relation from A to B because $R \not\subseteq A \times B$. [Since $(5, 7) \in R$ but $(5, 7) \notin A \times B$].

Domain and Range of a Relation

Domain of a relation

Let R be a relation from A to B. The domain of relation R is the set of all those elements $a \in A$ such that $(a, b) \in R$ for some $b \in B$.
Domain of R = set of first components of all the ordered pairs belonging to R.

Range of a relation

Let R be a relation from A to B. The range of R is the set of all those elements $b \in B$ such that $(a, b) \in R$ for some $a \in A$.
Thus range of $R = \{b \in B : (a, b) \in R \text{ for some } a \in A\}$.
Range of R = set of second components of all the ordered pairs belonging to R.

Co-domain of a relation

If R be a relation from A to B, then B is called the co-domain of relation R.
Clearly, range of a relation $\subseteq$ co-domain of relation.

Total Number of Relations

Let A and B be two non-empty finite sets having p and q elements respectively.
Then $n(A \times B) = n(A).\ n(B) = pq$
Therefore, total number of subsets of $A \times B = 2^{pq}$. Since each subset of $A \times B$ is a relation from A to B, therefore total number of relations from A to B is 2^{pq}
For example,
Let $A = \{1, 2\}$, $B = \{3, 4, 5\}$
Then $n(A \times B) = n(A).\ n(B) = 2.\ 3 = 6$
$\therefore$ Number of relations from A to B $= 2^6 = 64$

Illustration 11

Let A = {2, 3, 5} and B = {4, 7, 10, 8}. Find relation R : A → B such that a R b ⇒ a divides b. Also find domain, co-domain and range of relation.

$R = \{(2, 4), (2, 10), (2, 8) (5, 10)\}$
Here domain of $R = \{2, 5\}$
and range of $R = \{4, 10, 8\}$
Co-domain of $R = B = \{4, 7, 10, 8\}$

Illustration 12

Let A = {1, 2, 3}, B = {2, 4, 6, 8}, R_1 = {(1, 2), (2, 4), (3, 6)} and R_2 = {(2, 4), (2, 6), (3, 8), (1, 6)}. Check whether R_1 and R_2 are relations from A to B. If yes find domain and range of relations.

R_1 and R_2 are relations from A to B because $R_1 \subseteq A \times B$, and $R_2 \subseteq A \times B$
Here $1R_12$, $2R_14$, $3R_16$.
Also $2R_24$, $2R_26$, $3R_28$, $1R_26$.
Domain $R_1 = \{1, 2, 3\}$, Range $R_1 = \{2, 4, 6\}$
Domain $R_2 = \{2, 3, 1\}$, Range $R_2 = \{4, 6, 8\}$

Relation on a Set

A relation R from a non-empty set A to itself is called a relation on A. In other words if A is non-empty set, then a subset of $A \times A$ is called a relation on A.

For examples-

(i) Let A = {1, 2, 3} & for a, b $\in$ A, a R b $\Rightarrow$ a > b. Then, R = {(3,1), (3, 2), (2, 1)} is a relation on A.

(ii) Let A = {a, b, c} and R = {(a, a), (b, c), (c, a)}. Here $R \subseteq A \times A$ and therefore R is a relation on A.

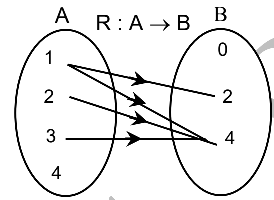
Arrow Diagram

In this form, the relation R is represented by drawing arrows from first components to the second components of all ordered pairs belonging to R.

For example : Let A = {1, 2, 3, 4}, B = {0, 2, 4} and R be the relation "is less than" from A to B, then

R = {(1, 2), (1, 4), (2, 4), (3, 4)}

The arrow diagram as shown in the given figure can represent this relation R from A to B.



Inverse Relation

If $R \subseteq A \times B$ be a relation from A to B. Then the Inverse relation of R, to be denoted by R^{-1}, is a relation from B to A defined by R^{-1} = {(b, a) : (a, b) $\in$ R}

Thus (a, b) $\in$ R $\Leftrightarrow$ (b, a) $\in R^{-1}$, $\forall$ a $\in$ A, b $\in$ B

Clearly domain of R^{-1} = Range of R and Range of R^{-1} = Domain of R

Illustration 13

Let A = {a, b, c} and B = {1, 2} and R = {(a, 1), (c, 1)}, find R^{-1}.

Given, R = {(a, 1), (c, 1)}

$\therefore$ R^{-1} = {(1, a), (1, c)}.

Illustration 14

Let N = Set of all natural numbers and R = {(x, y) : x + 2y = 0 ; y $\in$ N} . Is R a relation on N? Give reasons.

x + 2y = 0 $\Rightarrow$ x = –2y ; y $\in$ N.

Then x = –2, –4, –6, ……

Clearly (x, y) = (–2, 1) for y = 1.

So, (–2, 1) $\in$ R whereas (–2, 1) $\notin N \times N$.

$\therefore$ $R \not\subseteq N \times N$. Hence R is not a relation on N.

Illustration 15

Let R be a relation on set of natural numbers N defined by xRy $\Leftrightarrow$ x + 2y = 41; $\forall$x, y $\in$ N. Find the domain and Range of R.

x can be only those natural numbers for which y $\in$ N i.e. $y = \frac{41-x}{2} \in N$.

Clearly x = 1, 3, 5, 7, …., 39.

Similarly, y can be only those natural numbers for which x $\in$ N i.e., x = 41 – 2y $\in$ N.

Clearly, y = 1, 2, 3, …., 20

$\therefore$ Domain R = {x : (x, y) $\in$ R} = {x : x + 2y = 41}
= {1, 3, 5, 7, …., 39}
= set of odd natural numbers less than 40.

Range of R = {y : x + 2y = 41}
= {1, 2, 3, …, 20}
= set of natural numbers less than 21.

Illustration 16

Let R be the relation on Z defined by R = {(a, b) ; a, b ∈ Z, a – b is an integer}. Find the domain and range of R.

Here a, b are integers. Clearly a – b is an integer for all a, b ∈ Z.
Hence domain of R = Z
Also, range of R = Z

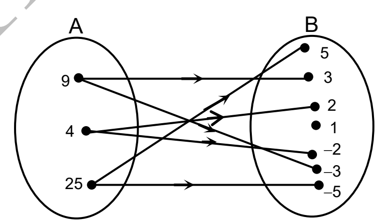
Illustration 17

Let A = {1, 2}. List all the relation on A.

Given A = {1, 2} ⇒ A × A = {(1, 1), (1, 2), (2, 1), (2, 2)}
Number of elements in A × A = 4
∴ Number of subsets of A × A = 2^4 = 16
∴ Number of relation on A = 16
All possible relations on set A are:
ϕ, {(1, 1)}, {(2, 2)}, {(1, 2)}, {(2, 1)}, {(1, 1), (2, 2)}, {(1, 1), (1, 2)}, {(1, 1), (2, 1)}, {(2, 2), (1, 2)}, {(2, 2), (2, 1)}, {(1, 2), (2, 1)}, {(1, 1), (2, 2), (1, 2)}, {(1, 1), (2, 2), (2, 1)}, {(1, 1), (1, 2), (2, 1)}, {(2, 2), (1, 2), (2, 1)}, A × A.

Illustration 18

Given figure shows a relation between the sets A and B. Write this relation
(i) in roster form
(ii) in set builder form.
What is its domain and range?

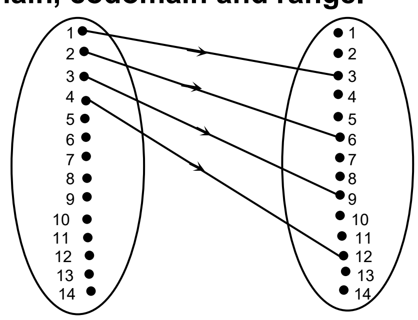


Let R be the given relation
In roster form
R = {(9, 3), (9, –3), (4, 2), (4, –2), (25, 5), (25, –5)}
In set builder form
R = {(x, y) : x ∈ A, y ∈ B and x is the square of y}
Domain of R = {9, 4, 25}
Range of R = {3, –3, 2, –2, 5, –5}

Illustration 19

Let A = {1, 2, 3, …,14}. Define a relation R from A to A by R = {(x, y) : 3x – y = 0, where x, y ∈ A} Depict this relation using an arrow diagram. Write down its domain, codomain and range.

Given, 3x – y = 0 ⇒ y = 3x
Domain of R = {1, 2, 3, 4}
Range of R = {3, 6, 9, 12}
Codomain of R = {1, 2, 3, …., 14}



Types of Relations

I. Void Relation

Let A be a set. Then ϕ ⊆ A × A and so it is a relation on A. This relation is called the void or empty relation on A.

II. Universal Relation

Let A be a set. Then A × A ⊆ A × A and so it is a relation on A. This relation is called the universal relation on A.

Note

The void and the universal relations on a set A are respectively the smallest and the largest relations on A.

III. Identity Relation

The identity relation on a set A is the set of ordered pairs belonging to A × A is denoted by I_A.

Symbolically, $I_A = \{(a, a) : a \in A\}$. For example: If $A = \{\alpha, \beta, \gamma\}$ then $I_A = \{(\alpha, \alpha), (\beta, \beta), (\gamma, \gamma)\}$

Illustration 20

Let A = {2, 3, 5}. Are the following identity relations?

(i) $R_1 = \{(2, 2), (5, 5)\}$,
(ii) $R_2 = \{(2, 3), (3, 3), (2, 2), (5, 5)\}$

(i) $R_1 = \{(2, 2), (5, 5)\}$ is not an identity relation on A because the element $(3, 3) \notin R_1$.

(ii) $R_2 = \{(2, 3), (3, 3), (2, 2), (5, 5)\}$ is not an identity relation on A because the element (2, 3) is not an element of identity relation.

Practice Assignment–II

1. If A = {1, 2, 3}, then write down the elements of the relation $R = \{(x, y) : x = y\}$ in A × A.

2. If A = {4, 5, 9} and B = {4, 6, 8}, then express a relation R in A × B if a < b, where $a \in A$ and $b \in B$

3. Find the domain and range of the relation R = {(x, y) : x + y = 8 and x, y are natural numbers}.

4. If N is the set of all natural numbers, then find the relation $R = \{(x, y) : x \in N, y \in N \text{ and } 2x + y = 10\}$ in N × N

5. If A = {1, 2, 3, 4}, B = {1, 2, 3, 4, 5, 6, 7} and $R = \{(x, y) : x \in A, y \in B \text{ and } y = x^2 - 3x + 3\}$, write all the elements of R.

6. Let R = {(1, 3), (2, 5), (3, 7), (4, 9), (5, 11)} be a relation in the set A = {1, 2, 3,…., 11}. Find the domain and range of R.

7. Let A = {1, 2, 3, …, 70} and R be the relation 'is square of' in A. Find R and its domain and range.

8. Let A = {1, 2, 3, …, 30} and R be the relation 'is one-fourth of' in A. Find R. Find also the domain and range of R.

9. Let R be a relation from A = {1, 2, 3} to B = {1, 2, 5, 6}. Write R as a set of ordered pairs when, (i) R is the relation 'is less than', (ii) R is the relation 'is greater than'

10. Find the domain and range of the following relations:

(i) $R = \{(x, x^2) : x \le 4, x \in N\}$

(ii) $R = \{(2x + 3, 1 + x) : 3 \le x \le 5, x \in N\}$

(iii) $R = \left\{\left(x+1, \frac{1+x^2}{1-x^2}\right) : 2 \le x \le 4, x \in N\right\}$

(iv) $R = \{(x + 1, x + 5) \mid x \in \{0, 1, 2, 3, 4, 5\}\}$

(v) $R = \{(x, x^3) \mid x$ is a prime number less than 10$\}$.

11. Let R be the relation on Z defined by $R = \{(a, b): a \in Z, b \in Z, a^2 = b^2\}$. Find (i) R, (ii) domain of R, (iii) range of R.

12. Let R be a relation from Q into Q defined by $R = \{(a, b): a, b \in Q \text{ and } a - b \in Z\}$. Show that

(i) $(a, a) \in R$ for all $a \in Q$,

(ii) $(a, b) \in R$ implies $(b, a) \in R$,

(iii) $(a, b) \in R$, $(b, c) \in R$ implies $(a, c) \in R$

13. Let R be a relation from N into N defined by R = {(a, b): a, b ∈ N and $a = b^2$}. Are the following statements true?
(i) (a, a) ∈ R, for all a ∈ N. (ii) (a, b) ∈R implies (b, a) ∈R.
(iii) (a, b) ∈R, (b, c) ∈ R implies (a, c) ∈ R?
Justify your answer in each case.

14. Let A = {1, 2, 3, 4} and B = {x, y, z}. Which of the following are relations from A to B.
(i) {(1, x), (2, x), (3, y), (4, z)} (ii) {(1, 1), (2, x), (3, y), (4, z)}
(iii) {(1, x), (2, y), (3, z), (4, z), (z, z)} (iv) A × B

15. Write down the elements of the following relations and also write their domain and range.
(i) A = {(x, y) : x, y ∈ N and x = 3y + 1} (ii) S = {(x, y) : x, y ∈ I and $x^2 + y^2 \le 9$}

16. Write the domain and range of the each of the following relations:
(i) {(x, 1/x) : 0 < x < 7, x is an integer} (ii) {(1, 1), (2, 4), (3, 9), ($\sqrt{5}$, 5)}
(iii) {(1/2, π/6), ($\sqrt{2}$ /2, π/4), ($\sqrt{3}$ /2, π/3), (1, π/2)}

17. In each of the following state which ordered pairs belong to the given relation:
(i) {(x, y) : x > y + 3} ; (1, 0), (2, 0), (3, 1), (4, 1) (6, 2), (7, 3)
(ii) {(x, y) : y = x + 1} ; (0, 1), (2, 2), (2, 3), (3, 4) (5, 7), (9, 8)
(iii) {(x, y) : $x^2 + y^2 = 25$} ; (–2, –3), (3, 4), (–3, 4), (3, –4), (–3, –4), (0, 5)

18. Write the elements of the relation R = {(x, y) : $x^2 + y^2 = 169$, x, y, ∈ I} in roster form

19. Let A = {(–2, –1, 0, 1, 2} and B = {0, 1, 3)}
(i) How many elements are there in A × B?
(ii) Write the elements of A × B.
(iii) Find the subset of A × B corresponding to the relation G : x > y
(iv) Find the subset of A × B corresponding to the relation L :| x | < y

20. Determine the set of ordered pairs satisfying the following relations
(i) R = {(a, b) : a ∈ N, a < 5, b = 4}
(ii) S = {(a, b) : b = |a – 1|, a ∈ Z and |a| ≤ 3}

21. I is the set of integers. Describe the following relation in set builder form, give its domain and range.
{(0, –7), (2, –5), (4, –3), (–13, –20), ….}

22. Let A = {x, y, z} and B = {a, b}. Find the total number of relations from A to B.

23. Define a relation R on the set N of natural numbers by
R = {(x, y) : y = x + 5, x is a natural number less than 4, x, y ∈ N}.
Depict this relationship using (i) roster form (ii) an arrow diagram.
Write down the domain and range on R.

24. Let A be the set of first five natural numbers and let R be relation on A defined as follows
(x, y) ∈ R ⇔ x ≤ y
Express R and R^{-1} as sets of ordered pairs. Determine also
(i) the domain of R^{-1}
(ii) the range of R.

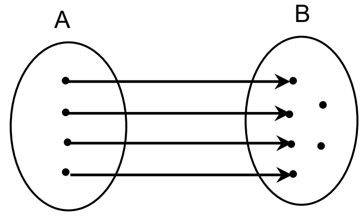
25. Find the inverse relation R^{-1} in each of the following cases :
(i) R = {(1, 2), (1, 3), (2, 3), (3, 2), (5, 6)}
(ii) R = {(x, y) : x , y ∈ N, x + 2y = 8}
(iii) R is relation from {11, 12, 13} to {8, 10, 12} defined by y = x – 3.

Functions

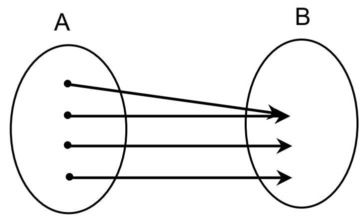
Before studying and knowing about function we read another classification of relations . These are :
(a) One-one
(b) Many-one
(c) One-many
(d) Many-many

Let us understand about these relations with the help of arrow diagram.

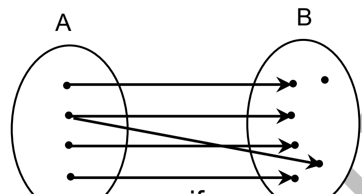
(a) A relation is said to be one-one if distinct elements of A have distinct image in B.



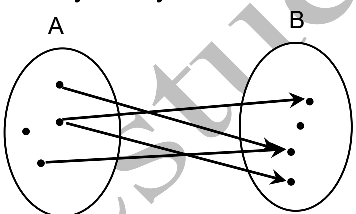
(b) A relation is said to be many-one if more than one elements of set A have a single image in B.



(c) A relation is said to be one-many if a single element of set A has more than one images in B.



(d) A relation is said to be many-many if more than one elements of set A are connected to multiple elements of set B.



The concept of a function plays an important role in mathematics. A function is an important type of relation as it also helps indicate the rule of association or correspondence between two elements or objects of two non-empty sets.

Function As a Relation

Let A and B be two non-empty sets. A relation f from A to B, i.e., a sub-set of A x B, is called a function (or a mapping or a map) from A to B if

(i) for each $a \in A$ there exists $b \in B$ such that $(a, b) \in f$.

(ii) $(a, b) \in f$ and $(a, c) \in f \Rightarrow b = c$.

Thus, a non-void subset f of A x B is a function from A to B if each element of A appears in some ordered pair in f and no two ordered pairs in f have the same first element.
Clearly, of the four classifications of relations given above only first two are functions, while last two are mere a relation. Hence it must be noted that all the functions are relations but each relation can not be a function.

Difference between Relation and Function.

1. If R is a relation from A to B, then domain of R may be a subset of A. But if f is a function from A to B, then domain f is equal to A.

2. In a relation from A to B an element of A may be related to more than one element in B. But in a function from A to B each element of A must be associated to one and on y one element of B.

Definition

Let A and B be two non-empty sets. If these exists a rule of correspondence by which each element $x \in A$ is related to a unique element $y \in B$, then such correspondence is called a function from A to B. It is written as

$f : A \to B$ such that $x \in A$ and $y \in B$, where $y = f(x)$

If f is a function from A into B, then we write $(x, y) \in f$ as $f(x) = y$, where $x \in A$, $y \in B$. The element $y \in B$ is called the **image** of x under f and x is called the **pre-image** of y under f.

Note : In a function $f : X \to Y$,

1. Each element of the set X must be associated with a unique element of Y.

2. Of course two or more elements of the set X may be associated with the same element of Y.

3. There may be some elements of the set Y which are not assigned to any element of the set X.

Remarks.

1. **Value of a function :** The element $y \in B$ that is associated to x under f is denoted by f(x) and is called the value of f at x.

2. **Domain of f :** The set A is called the domain of function f.

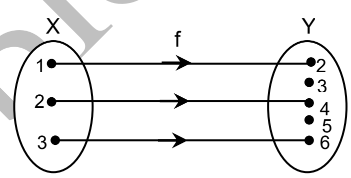
3. **Co-domain of f :** The set B is called the co-domain of function f.

4. **Range of f :** The set $\{f(x) : x \in A \text{ and } f(x) \in B\}$ for all values taken by f is called range of f. Obviously , it is subset of set B.

Illustration 21

Let X = {1, 2, 3}, Y = {2, 3, 4, 5, 6} and let f be a relation defined by f(1) = 2, f(2) = 4, f(3) = 6. Is it a function ?

Fig. Represents two sets and the correspondence between their elements



We observe that every element of X has an image in Y (or equivalently, we do not have any element in X which does not have an image in Y) and image of each element of X is unique (or equivalently no element of X has more than one image)

Thus, we may say that $f : X \to Y$ is a function.

Illustration 22

Let X = {a, b, c}, Y = {2, 3, 4, 5} and f be the relation defined by f(a) = 2, f(b) = 4. Is f a function from X to Y

Here f is not a function as $c \in X$ does not have its image .

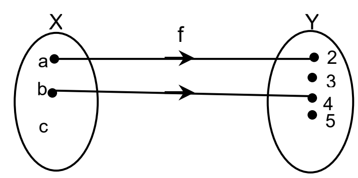


Illustration 23

Let X = {a, b, c} and Y = {2, 3, 4, 5} and let f be a relation defined by f(a) = 2, f(b) = 3, f (b) = 4 , f(c) = 5. Is it a function ?

Here every element of X has an image in Y but there is one element $b \in X$ which does not have unique image. Here 'b' has two images. Hence f is not a function.

Illustration 24

Let A = {1, 2, 3, 4} and B = {a, b, c}

Let R = {(1, a), (2, a), (3, b), (4, b)}. Then R is a function from A to B. Obviously, R is also a relation from A to B because $R \subseteq A \times B$. But consider the subset S of $A \times B$ given by S = {(1, a), (2, b), (1, c), (4, b)}. Here, S is a relation from A to B because $S \subseteq A \times B$. But S is not a function from A to B. The obvious reason is that the element $1 \in A$ is associated to two different elements a and $c \in B$

Illustration 25

Let A = {–2, –1, 0, 1, 2} and $f : A \to Z$ given by $f(x) = x^2 - 2x - 3$. Find (a) the range of f (b) pre-image of 6, –3 and 5.

(a) We have : $f(-2) = (-2)^2 - 2(-2) - 3 = 5$, $f(-1) = (-1)^2 - 2(-1) - 3 = 0$

$f(0) = -3$, $f(1) = 1^2 - 2 \times 1 - 3 = -4$ and $f(2) = 2^2 - 2 \times 2 - 3 = -3$.

So range (f) = {0, 5, –3, –4}

(b) Let x be the pre-image of 6, Then,

(c) $f(x) = 6 \Rightarrow x^2 - 2x - 3 = 6 \Rightarrow x^2 - 2x - 9 = 0 \Rightarrow x = 1 \pm \sqrt{10}$

Since $x = 1 \pm \sqrt{10} \notin A$. So, there is no pre-image of 6.

Let x be the pre-image of –3. Then,

$f(x) = -3 \Rightarrow x^2 - 2x - 3 = -3 \Rightarrow x^2 - 2x = 0$

$\Rightarrow x = 0, 2$

Since, $0, 2 \in A$, so 0, 2 are the pre-images of –3.

Let x be the pre-image of 5. Then,

$f(x) = 5 \Rightarrow x^2 - 2x - 3 = 5 \Rightarrow x^2 - 2x - 8 = 0$

$\Rightarrow (x - 4)(x + 2) = 0 \Rightarrow x = 4, -2$

Since, $-2 \in A$, So, –2 is the pre-image of 5.

Illustration 26

Express the following functions as sets of ordered pairs and determine their ranges.

(a) $f : A \to R$, $f(x) = x^2 + 1$, where A = {–1, 0, 2, 4}

(b) $g : A \to N$, $g(x) = 2x$, where $A = \{x : x \in N, x \le 10\}$

(a) We have, $f(-1) = (-1)^2 + 1 = 2$

$f(0) = 0^2 + 1 = 1$, $f(2) = 2^2 + 1 = 5$ and $f(4) = 4^2 + 1 = 17$

So, $f = \{x, f(x)) : x \in A] = \{(-1, 2), (0, 1), (2, 5), (4, 17)\}$

So, Range of (f) = {2, 15, 17}

(b) We have, A = {1, 2, 3, …10}. So,
$g(1) = 2 \times 1 = 2$, $g(2) = 2 \times 2 = 4$, $g(3) = 2 \times 3 = 6$, $g(4) = 2 \times 4 = 8$.
$g(5) = 2 \times 5 = 10$, $g(6) = 2 \times 6 = 12$, $g(7) = 2 \times 7 = 14$, $g(8) = 2 \times 8 = 16$.
$g(9) = 2 \times 9 = 18$ and $g(10) = 2 \times 10 = 20$
$\therefore g = \{(x, g(x) : x \in A\} = \{(1, 2), (2, 4), (3, 6), \ldots, (10, 20)\}$
Range of $g = g(A) = \{g(x) : x \in A\} = \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20\}$.

Illustration 27

Let f : {(1, 1), (2, 3), (0, –1), (–1, –3)} be function described by the formula f(x) = ax + b for some integers a, b. Determine a, b.

We have , $f(1) = 1$, $f(2) = 3$, $f(0) = -1$ and $f(-1) = -3$. It is given that $f(x) = ax + b$. Therefore,
$f(1) = 1$ and $f(2) = 3 \Rightarrow a + b = 1$ and $2a + b = 3 \Rightarrow a = 2, b = -1$
Thus, $f(x) = 2x - 1$. Clearly, $f(0) = -1$ and $f(-1) = -3$ are true.
Hence , $a = 2$, $b = -1$.

Equal Functions

Two functions f and g are said to be equal iff
(i) the domain of f = domain of g,
(ii) the co-domain of f = the co-domain of g, and
(iii) f(x) = g(x) for every x belonging to their common domain
If two function f and g are equal, then we write f = g.

Illustration 28

Let f: R – {2} → R be defined by $f(x) = \dfrac{x^2 - 4}{x - 2}$ and g : R → R be defined by g(x) = x + 2. Find whether f = g or not.

Clearly, $f(x) = g(x)$ for all $x \in R - \{2\}$. But, f(x) and g(x) have different domains. In fact, domain of f = R – {2} and domain of g = R. Therefore, $f \neq g$.

Illustration 29

Let f : Z → Z , g : Z → Z be function defined by $f = \{(n, n^2) : n \in Z\}$ $g = \{(n, |n|^2) : n \in Z\}$. Show that f = g.

Clearly, domain of f = Domain of g = Z and , Co-domain of f = Co-domain of g = Z.
We have , $f(n) = n^2$ and $g(n) = |n|^2 = n^2$. $[\because |n|^2 = n^2]$
$\therefore f(n) = g(n)$ for all $n \in Z$.
Hence , f = g.

Illustration 30

Find the domain for which the functions $f(x) = 2x^2 - 1$ and $g(x) = 1 - 3x$ are equal.

We have $f(x) = g(x) \Rightarrow 2x^2 - 1 = 1 - 3x \Rightarrow 2x^2 + 3x - 2 = 0$
$\Rightarrow (x + 2)(2x - 1) = 0 \Rightarrow x = -2, 1/2$.
Thus , f(x) and g(x) are equal on the set {–2, 1/2}

Practice Assignment–III

1. Define function as a relation from one set to another.

2. Can any relation be a function? Justify your answer along with illustrations.

3. If A = {– 1, 0, 1, 2} and f : A → Z be a function defined by f(x) = 2x + 3, find (i) the range of f, (ii) the pre-images of 5 and 7.

4. If A = {1, 2, 3}, B = {4, 5}, then state which one of the following is a function:
(i) {(1, 4), (2, 5), (3, 4)}; (ii) {(1, 5), (2, 4), (2, 5)};
(iii) {(2, 4), (1, 5), (3, 4), (4, 2)}; (iv) {(3, 4), (2, 4), (1, 5)}.

5. Draw arrow diagram for each of the following functions:
(i) {(1, 3), (2, 4), (3, 2)}; (ii) {(2, 4), (3, 1), (5, 3), (1, 4)}
(iii) {(1, 2), (2, 7), (3, 5), (5, 2), (6, 3)}.

6. If a function f: Z → Z is defined by f(x) = $\begin{cases} 2x-3 & \text{if } x<0 \\ 5 & \text{if } x=0 \\ 5x+7 & \text{if } x>0 \end{cases}$, find f(0), f(–2) and f(3), where Z is the set of integers.

7. If A = {1, 2, 3}, B = {4, 5} and f = {(1, 4), (2, 5)}, then examine whether f is a function from A into B. If not, how can you change A for f to be a function?

8. Let A = {–3, –2, 1, 3}, B = {–9, –7, –3, 11} and f(x) = x^2 + 3x – 7.
Find f(A) and show that f(A) = B = range of f. What is the image of –2under f and what is the pre-image of –3?

9. Let f: R → R be given by f(x) = 4x – 3. Find (i) {x : f(x) = 21}, (ii) the pre-image of 13 and 97.

10. Let f be the relation on the set N of all natural numbers defined by f = {(x, 3x): x ∈ N}. Is f a function from N into N? if so, find the range of f.

11. Express each of the following functions as sets ordered pairs and hence find its range, where Q is the set of all rational numbers:
(i) f : A → Q, f(x) = $\frac{2x}{3x+5}$, where A = {–1, 1, 3, 4}
(ii) f : A → Q, f(x) = $\frac{2x+1}{5x+3}$, where A = {0, 1, 2, 3}

12. Express the following function as the set of ordered pairs and hence find the range of the function:
f : A → Q, f(x) = $\frac{2x-1}{3x+2}$, where | x | ≤ 4, x ∈ Z, Z being the set of all integers and Q, the set of all rational numbers.

13. Given that f(x) = x + 1, φ(x) = $\frac{x^2-1}{x-1}$. Is f(x) = φ(x) ? Give reasons.

14. If f(x) = x^2, find $\frac{f(1\cdot1)-f(1)}{1\cdot1-1}$.

15. The function 't' maps the temperature in Celsius into temperature in Fahrenheit. It is defined by
t(C) = $\frac{9C}{5}$ + 32. Find
(i) t(0)
(ii) t(28)
(iii) t(–10)
(iv) value of C when t(C) = 212

16. Let A = {9, 10, 11, 12, 13} and let f : A → N be defined by
f(n) = highest prime factor of n. Find the range of f.

17. Let f be a subset of Q × Z defined by f = $\left\{\left(\frac{m}{n}, m\right) : m \in Z, n \in Z, n \neq 0\right\}$. Is f a function from Q into Z ? Justify your answer.

18. Let f be a subset of $Z \times Z$ defined by $f = \{(ab, a + b) : a, b \in Z\}$. Is f a function from Z into Z ? Justify your answer.

Real Functions

If the domain and co-domain of a function are subsets of R (set of all real numbers). It is called a real valued function or in short a real function.

Description of a real function

If f is a real valued function with finite domain, then f can be described by listing the values, which it attains at different points of its domain. However, if the domain of a real function is an infinite set, then f cannot be described by listing the values at points in its domain. In such cases real functions are generally described by some general formula or rule like $f(x) = x^2 + 1$ or $f(x) = 2 \sin x + 3$ etc. In calculus almost all real functions are described by some general formula or rule.

Total Number of Functions

Let A and B be two finite sets having m and n elements respectively. Then each element of set A can be associated to any one of n elements of set B. So, total number of functions from set A to set B is equal to the number of ways of doing m jobs where each job can be done in n ways. The total number of such ways is $n \times n \times n \ldots\ldots \times n$ (m–times).

Hence, the total number of functions from A to B is n^m i.e. $[n(B)]^{n(A)}$.

Finding of Domain and Range of real functions

Domain of real function

Real functions in calculus are described by some formula and their domains are not explicitly stated. In such cases to find the domain of a function f (say) we use the fact that the domain is the set of all real numbers x for which f(x) is a real number.

Range of a real function

The range of a function f(x) is the set of values of f(x) which it attains at points in the domain. For a real function the co-domain is always a subset of R. So, range of a real function f is the set of all points y such that $y = f(x)$, where $x \in$ Dom f(x). In order to find the range of real function proceed as follows -

(i) Put $f(x) = y$

(ii) Solve the equation in step (i) for x to obtain $x = \phi(y)$.

(iii) Find the values of y for which the values of x, obtained from $x = \phi(y)$ are in the domain of f

(iv) The set of values of y obtained in step (iii) is the range of f.

Illustration 31

If $f(x) = x^2 - x$, then prove that $f(h + 1) = f(-h)$.

We have $f(x) = x^2 - x$

$\therefore$ $f(h + 1)^2 = (h + 1)^2 - (h + 1) = h^2 + 2h + 1 - h - 1 = h^2 + h$ and $f(-h) = (-h)^2 - (-h) = h^2 + h$

Hence $f(h + 1) = f(-h)$.

Illustration 32

If $f(x) = e^{px + q}$ where p, q, are constants, show that $f(a).f(b).f(c) = f(a + b + c)e^{2q}$.

We have $f(x) = e^{px + q}$

$\therefore$ $f(a) = e^{pa + q}$, $f(b) = e^{pb + q}$ and $f(c) = e^{pc + q}$

$\therefore$ L.H.S.= $f(a).f(b).f(c) = e^{pa + q} \times e^{pb + q} \times e^{pc + q}$

$= e^{pa + pb + pc + 3q} = e^{p(a + b + c) + 3q}$.

R.H.S. = $f(a + b + c) . e^{2q}$

$= e^{p(a+b+c)+q} . e^{2q} = e^{p(a+b+c)+3q}$

Hence L.H.S. = R.H.S.

Illustration 33

Find domain and range of $f(x) = \dfrac{X}{1 - X}$

Clearly f(x) is not defined at x = 1 Hence, domain of f is R – {1}

Put $\quad y = f(x) = \dfrac{x}{1-x}$

or $\quad x = \dfrac{y}{y+1}$

Now, x will exist if $y \neq -1$. Hence, range of f is R – {–1}

Illustration 34

Find domain and range of $f(x) = \sqrt{9 - x^2}$

For f(x) to define, $9 - x^2 \geq 0$ or $\quad x \in [-3, 3]$

Put $y = f(x) = \sqrt{9 - x^2}$ or $\quad x = \pm\sqrt{9 - y^2}$

For x to exist, $9 - y^2 \geq 0$ or $\quad y \in [-3, 3]$

But $y \geq 0$, Hence, range of f is [0, 3]

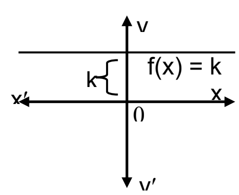
Graph of a Real Function

The graph of a real function f is the graph of the equation y = f(x), where x ∈ domain of f. Clearly, every value of x ∈ domain corresponds to a unique value of y in the range.
To draw the graph of a function, we take two rectangular axes X′OX and Y′OY, O being the origin, and then plot all the corresponding pairs of values of x and y as points on the plane of the axes (Plane of the paper)
In practical problems, we first tabulate a number of corresponding pairs of values of x and y for the given function and then plot them as point in the cartesian (plane of the paper) referred to two rectangular axes.

Some Real Functions

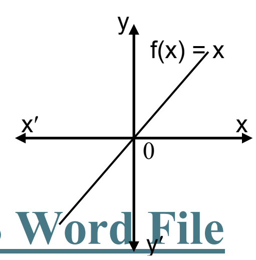
(i) Constant function

Let k be a fixed real number. Then a function f(x) given by f(x) = k for all x ∈ R is called a constant function.



The domain of the constant function f(x) = k is the complete set of real numbers and the range of f is the singleton set {k}

(ii) Identity function



The function defined by f(x) = x for all $x \in R$, is called identity function on R. The domain and range of the Identity function are both equal to R.

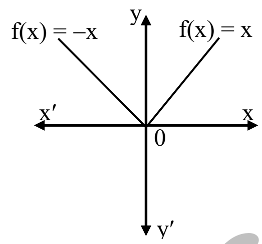
(iii) Modulus function

The function defined by

$$f(x) = |x| = \sqrt{x^2} = \max\{x, -x\} = \begin{cases} x, & \text{when } x \geq 0 \\ -x, & \text{when } x < 0 \end{cases}$$ is

called the modulus function.

The domain of the modulus function is the set of all real numbers and the range is the set of all nonnegative real numbers or $[0, \infty)$.



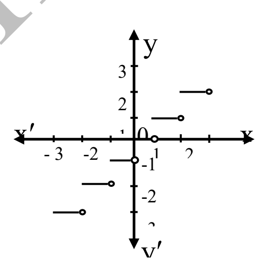
(iv) The greatest integer function

For any real number x, we denote by [x], the greatest integer less than or equal to x. For example,

[2.45] = 2, [–2.1] = –3, [1.75] = 1, [0.32] = 0 etc.

The function f defined by f(x) = [x] for all $x \in R$, is called the greatest integer function.

The domain of the greatest integer function is the set R of all real numbers and the range is the set of all Integers as it attains only integral values.

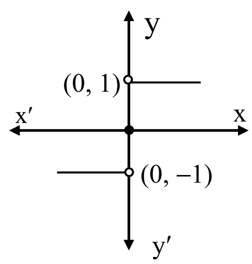


(v) Signum function

The function defined by $f(x) = \begin{cases} \dfrac{|x|}{x}, & x \neq 0 \\ 0 & x = 0 \end{cases}$ or

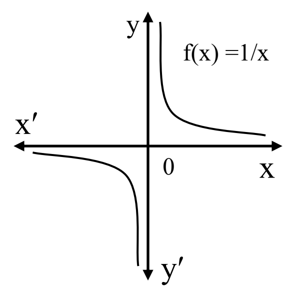
$$f(x) = \begin{cases} 1, & x > 0 \\ 0, & x = 0 \\ -1, & x < 0 \end{cases}$$ is called the signum function.

The domain of the signum function is R and the range is the set {–1, 0, 1}.

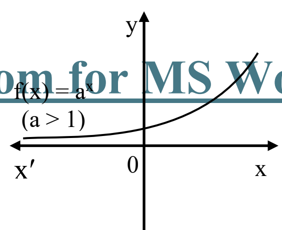


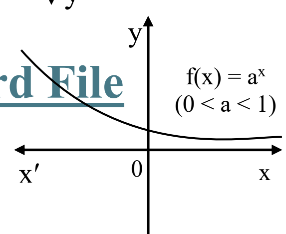
(vi) Reciprocal function

The function that associates each non-zero real number x to its reciprocal $\frac{1}{x}$ is called the reciprocal function. The domain and range of the reciprocal function are both equal to R – {0} i.e. the set of all non-zero real numbers.



(vii) Exponential function

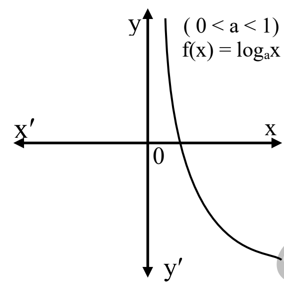


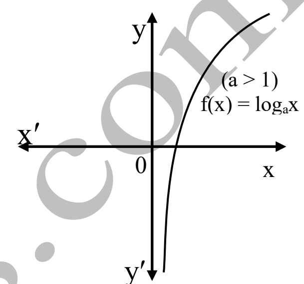


If a is positive real number and $a \neq 1$, then the function which associates every real number x to a^x i.e. $f(x) = a^x$ $(a > 0)$ is called the exponential function.
The domain of the exponential function is R and the range is the set of all positive real numbers.

(viii) Logarithmic function

If 'a' is a positive real number and $a \neq 1$, then the function that associates every positive real number to $\log_a x$ i.e. $f(x) = \log_a x$ is called the logarithmic function. The domain of the logarithmic function is the set of all positive real numbers and the range is the set R of all real numbers.

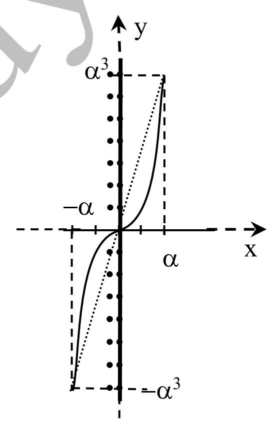




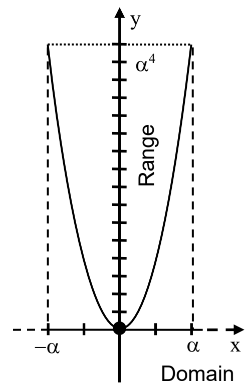
(ix) Polynomial function

A function of the form
$f(x) = a_0 x^n + a_1 x^{n-1} + \ldots\ldots + a_{n-1} x + a_n$, where $a_0, a_1, a_2, \ldots\ldots a_n$ are real numbers, $a_0 \neq 0$ and $n \in N$, is called polynomial function of degree n. The domain of a polynomial function is always R.
For Example – Domain and range of cubic function i.e. $f(x) = x^3$ and biquadratic function $f(x) = x^4$ are shown in the figure.

f : R → R; f(x) = x³

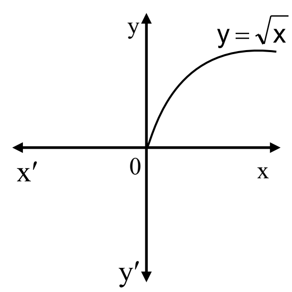


f : R → R; f(x) = x⁴



(x) Square root function

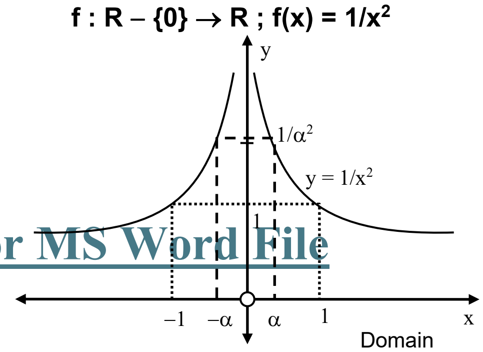
The function that associates every positive real number x to $+\sqrt{x}$ is called the square root function, i.e., $f(x) = +\sqrt{x}$
Domain of f is the set of all non-negative real numbers = $[0, \infty)$ and range (f) = $[0, \infty)$.



(xi) Rational function

A function of the form $f(x) = \dfrac{p(x)}{q(x)}$, where p(x) and q(x) are polynomial and $q(x) \neq 0$, is called a rational function.

f : R – {0} → R ; f(x) = 1/x²

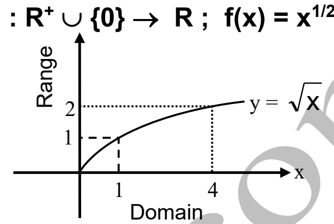


The domain of a rational function $\frac{p(x)}{q(x)}$ is the set of all real numbers except points where q(x) = 0.

(xii) Irrational functions

A function containing one term having non-integral rational powers of x are called Irrational functions. For examples $f(x) = \sqrt{x}$, $f(x) = x^{1/3}$ are irrational functions.

$f : R^+ \cup \{0\} \to R$; $f(x) = x^{1/2}$



LOGARITHMIC VALUES AND THEIR PROPERTIES

Let there be a number a > 0 and a ≠ 1. A number x is called the logarithm of another number b > 0 to the base a if $a^x = b$ and we write it as $x = \log_a b$.

Properties of log expressions : For a > 0, a ≠ 1, m > 0, n > 0 we have –

(i) $\log_a mn = \log_a m + \log_a n$

(ii) $\log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n$

(iii) $\log_a(m^n) = n \log_a m$

(iv) $\log_a m = \left(\frac{\log_b m}{\log_b a}\right)$

(v) $\log_b a = \frac{1}{\log_a b}$

(vi) $\log_a 1 = 0$

(vii) $\log_a a = 1$

(viii) $a^{\log_a n} = n$

(ix) $\log_a x < \log_a y \Leftrightarrow \begin{cases} x < y & \text{where } a > 1 \\ x > y & \text{where } 0 < a < 1 \end{cases}$

(x) $\log_a x = \log_a y \Leftrightarrow x = y$

Illustration 35

Draw the graph of x = 2 , the domain of definition being the set of all real numbers.

Here x = 2 and y has any value in $-\infty < y < \infty$. We have

x	2	2	2	2	2	.....
y	0	1	2	–1	–2	.....

When the corresponding pairs of values of x and y are plotted as points in the plane of OX and OY, we get an unbroken line parallel to y-axis at a distance of 2 units from it. This line is the required graph.

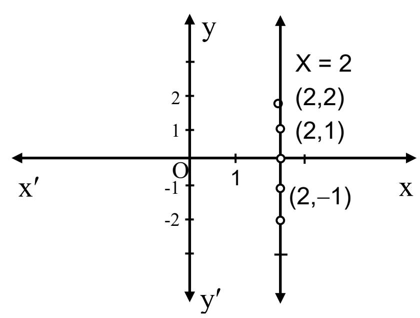


Illustration 36

Draw the graph of function f : R → R defined as follows :

f(x) = 3 + 2x, when x ≤ 0

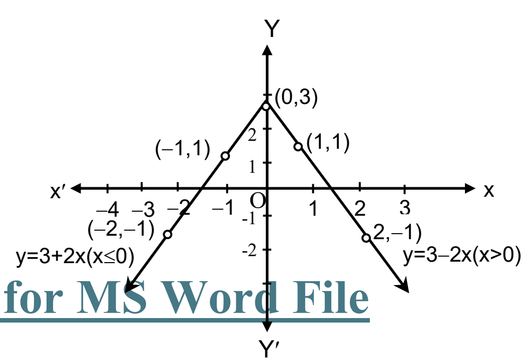
= 3 – 2x, when x > 0

Taking y = f(x), we have

y = 3 + 2x, when x ≤ 0 ...(1)

and y = 3 – 2x, when x > 0 ...(2)

in (1), we have,



x	0	–1	–2
y	3	1	–1

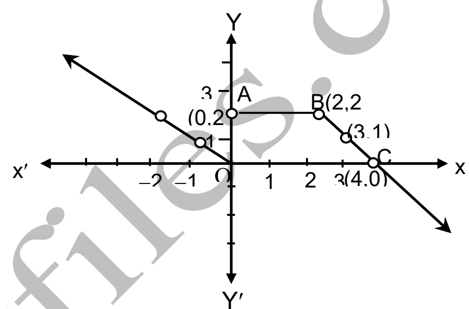
The straight line joining (0, 3) and (–1, 1) is drawn. The other points of (1) lie on this line.
From equation (2) we have,

x	1/2	1	2
y	2	1	–1

The straight line joining (1, 1) and (2, –1) is drawn. The other points of (2) lie on this line. The graph of the given function is shown in fig.

Illustration 37

Draw the graph of the function f : R → R defined by
$f(x) = -x$, if $x < 0$;
$= 2$, if $0 \le x < 2$;
$= 4 - x$, if $x \ge 2$.



We have
$y = f(x) = -x$, if $x < 0$
$= 2$, if $0 \le x < 2$
$= 4 - x$, if $x \ge 2$.
The graph of $y = -x$ if $x < 0$ is a straight line in the second

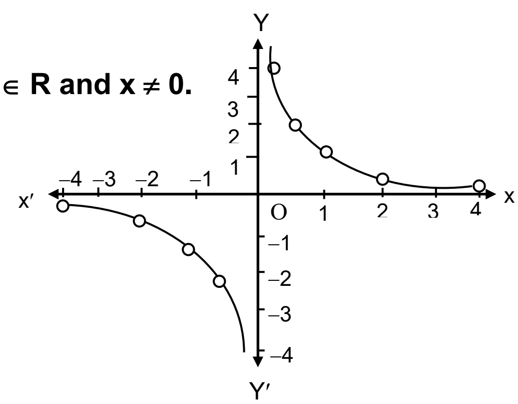
quadrant bisecting $\angle X'OY$.
$y = 2$ if $0 \le x < 2$ represents the line-segment AB in the first quadrant.
For $y = 4 - x$ if $x \ge 2$, we have

x	2	2.5	3	4
y	2	1.5	1	0

and the graph of this part is a straight line in the first and the 4th quadrant as shown in fig.

Illustration 38

Draw the graph of the function f : R → R define by $f(x) = \frac{1}{x}$, $x \in R$ and $x \neq 0$.



Let y = f(x) ; then $y = \frac{1}{x}$, We see that

(i) if $x > 0$, $y > 0$ and if $x < 0$, $y < 0$;
(ii) If x increases , y decreases and vice versa ;
(iii) The graph passes through the points (1, 1) and (–1, –1)

We conduct a table showing the values of x and the corresponding value of y as follows

	¼	1/2	1	2	3	4
y	4	2	1	1/2	1/3	¼

x	–1	–1/2	–1/4	–2	–4
y	–1	–2	–4	–1/2	–1/4

As $x \to 0^+$, $y \to \infty$ and as $x \to \infty$, $y \to 0$; again as $x \to 0^-$, $y \to -\infty$, and as $x \to -\infty$, $y \to 0$]
Thus we get two branches of the curve, one in the first quadrant and the other in third quadrant. As we take positive value of x nearer and nearer to zero, the values of y become larger and larger and ultimately infinitely large. Again when x takes negative values nearer and nearer to zero, the values of y become numerically larger and larger ultimately infinitely large.

Illustration 39

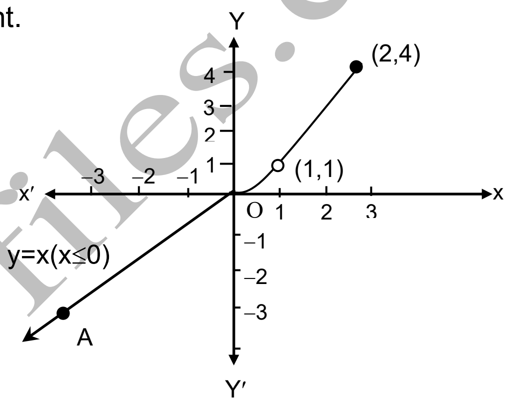
Draw the graph of the function f defined by $f(x) = x$ if $x \le 0$, $f(x) = x^2$ if $0 < x \le 2$.

We have

$y = f(x) = x$, if $x \le 0$ ....(1)

and $y = x^2$ if $0 < x \le 2$ ....(2)

We see that graph passes through the origin O (0, 0) and it consists of two parts : (i) a straight line $\overrightarrow{OA}$ in the third quadrant, (ii) a portion of a parabola in the first quadrant.



From (1) :

x	0	–1	–2	–3	…..
y	0	–1	–2	–3	…..

The points (0,0), (–1, –1), (–2,–2), (–3,–3), …. are plotted and joined and extended to obtain the line $\overrightarrow{OA}$.

From (2) :

x	0.5	1	1.5	2
y	0.25	1	2.25	4

(0.5, 0.25), (1,1) (1.5, 2.25), (2,4) are plotted and joined to obtain the parabolic portion in the first quadrant. The graph of the given function is shown in Fig.

Practice Assignment–IV

1. Name the function f defined by f(x) = $\frac{x^2 - 3x + 5}{2x + 7}$ and state its domain.

2. If f(x) = [x], when [x], denotes the greatest integer less than or equal to x, find f(0), f(3), f$\left(\frac{1}{2}\right)$, f(7/5), f(5.02), f(–3), f(–2.3).

3. If a function f is given by f(x) = |x| – 2x, find f(–1) and f(1)

4. If a function f : R → Q is given by f(x) = |x| – [x], then find the value of f(3.5) and f(–3.5)

5. Find the range of

(i) $f(x) = 2 - 3x, x \in R, x > 0$

(ii) $f(x) = x^2 + 2, x \in R$

6. Let f = $\left\{\left(x, \frac{2x}{2x+1}\right) : x \in R, x \neq -\frac{1}{2}\right\}$ be a function from R into R. Find the range of f.

7. Let f = $\left\{\left(x, \frac{3x^2}{1+x^2}\right) : x \in R\right\}$ be a function from R into R. Find the range of f.

8. Find the domain and range of real function f defined by

(i) f(x) = $\frac{x^2 - 1}{x - 1}$

(ii) f(x) = – |x|

(iii) f(x) = $\frac{1}{1 - x^2}$

(iv) f(x) = $\sqrt{x - 1}$

(v) f(x) = |x – 1|

(vi) f(x) = $\frac{x - 1}{|x - 1|}$

9. Find the domain of the following functions

(i) f(x) = $\frac{x^2 + 2x + 1}{x^2 - 8x + 12}$

(ii) f(x) = $\frac{x^2 + 3x + 5}{x^2 - 5x + 4}$

10. Find the domain of

(i) $\log_{10} |4 - x^2|$

(ii) $\log_e \left(\frac{2 + x}{2 - x}\right)$

11. Draw the graph of the function f defined by y = f(x) given by

(i) y = 6 ; (ii) y = –x ; (iii) y = 2x ; (iv) $\frac{x}{3} + \frac{y}{2} = 1$

12. Draw the graph of each of the function f defined by

(i) f(x) = $\begin{cases} 2, \text{ if } x > 0 \\ 0, \text{ if } x = 0 \\ -2, \text{ if } x < 0 \end{cases}$; (ii) f(x) = $\begin{cases} x \text{ if } x \geq 0 \\ -x \text{ if } x < 0 \end{cases}$

13. Draw the graph of the function f defined by

(i) $f(x) = \begin{cases} 2x \text{ if } x \geq 0 \\ -2x \text{ if } x < 0 \end{cases}$ (ii) $f(x) = \begin{cases} x, \text{if } x \leq 0 \\ -x, \text{if } 0 < x < 3 \end{cases}$

14. Draw the graph of the function f defined by

$$f(x) = \begin{cases} x-1, \text{if } x > 0 \\ -\frac{1}{2}, \text{if } x = 0 \\ x+1, \text{if } x < 0 \end{cases}$$

15. A function $f : R \to R$ is defined by $f(x) \begin{cases} 1+2x, + \text{if } x \leq 1 \\ 3-x, \text{if } x > 1 \end{cases}$. Draw the graph of the function.

16. Draw the graph of the function $f : R \to R$ defined by $f(x) = |x-4| - 2$

17. Draw the graph of the function $f : R \to R$ defined by

(i) $f(x) = \frac{|x|}{x}$; (ii) $g(x) = \frac{x}{|x|}$

18. Draw the graph of the function $f : R \to R$ defined by

(i) $f(x) = \frac{x^2}{x}$;

(ii) $g(x) = x$. Point out the difference between the two graphs.

19. Draw the graph of the function $f : R \to R$ defined by $f(x) = \begin{cases} 2x+1, \text{if } x \geq 1 \\ 2x-1, \text{if } x < 1 \end{cases}$

20. Draw the graph of $f : R \to R$ defined by $f(x) = x - [x]$. Find its domain and range.

Algebra of real functions

We shall now discuss how to add two real functions, subtract one real function from another, multiply a real function by a scalar (i.e., by a real number) or by another real function and divide a real function by another.

(i) **Addition of two Real Functions**

Let $f : X \to R$ and $g : X \to R$ be any two real functions, where $X \subset R$.
Then $(f + g) : X \to R$ define by $(f + g)(x) = f(x) + g(x)$, for all $x \in X$.

(ii) **Subtraction of a Real Function from another Function**

Let $f : X \to R$ and $g : X \to R$ be any two real functions, where $X \in R$.
Then $(f - g) : X \to R$ is defined by $(f - g)(x) = f(x) - g(x)$ for all $x \in X$.

(iii) **Multiplication of a Function by a scalar**

Let $f : X \to R$ be a real function and k be a scalar (i.e., a real number). Then $k f(x)$ is a real function from X to R defined by $(kf) x = k f(x)$ for all $x \in X$.

(iv) **Multiplication of two Real Functions**

The product of two real functions $f : X \to R$ $g : X \to R$ is a real function $fg : X \to R$ define by $(fg)\, x = f(x) \,.\, g(x)$ for all $x \in X$.

(v) **Quotient of two Real Functions**

Let f and g be two real functions defined by $f : X \to R$ and $g : X \to R$, where $X \subset R$. Then the quotient of f and g, say $\frac{f}{g}$ is a function defined by $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$, provided $g(x) \neq 0$, for $x \in X$.

Note : If f and g are two real valued functions with domains D_f and D_g respectively, then the above operations are defined for only $x \in D_f \cap D_g$.

Illustration 40

If $f(x) = x^2 + 2$ and $g(x) = 2x + 3$ be two real functions, then find f + g, f – g, fg and f/g.

(i) $(f + g)(x) = f(x) + g(x) = (x^2 + 2) + (2x + 3) = x^2 + 2x + 5$

(ii) $(f – g)(x) = f(x) – g(x) = x^2 + 2 – (2x + 3) = x^2 – 2x – 1$.

(iii) $(fg)(x) = f(x)\, g(x) = (x^2 + 2)(2x + 3) = 2x^3 + 3x^2 + 4x + 6$

and (iv) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{x^2+2}{2x+3}$, where $x \neq -\frac{3}{2}$.

Practice Assignment–V

1. If $f(x) = 2x^2 + 3x$ and $g(x) = 3x + 4$ be two real functions, find
(i) $(f + g)(x)$, (ii) $(2f + 3g)(x)$;
(iii) $(f – g)(x)$; (iv) $(fg)(x)$.

2. If $f(x) = \sqrt{x}$ and $g(x) = x$ be two functions defined over non-negative real numbers, then find $(f + g)(x)$, $(f – g)(x)$, $(fg)(x)$ and $\left(\frac{f}{g}\right)(x)$.

3. If $f(x) = e^{ax+b}$, prove that $e^b f(x + y) = f(x).f(y)$.

4. If $f(x) = a\left(\frac{x-b}{a-b}\right) + b\left(\frac{x-a}{b-a}\right)$, find f(a), f(b) and f(a + b) and hence verify that $f(a + b) = f(a) + f(b)$.

5. If $f(x) = \frac{ax+b}{bx+a}$, prove that $f(x).f\left(\frac{1}{x}\right) = 1$.

6. If $f(x) = \frac{x-1}{x+1}$, then show that $\frac{f(a)-f(b)}{1+f(a)f(b)} = \frac{a-b}{1+ab}$.

7. If $f(x) = e^x$, $g(x) = \log x$, find

(i) (f + g) (1) (ii) (f – g) (1) (iii) (fg) (1) (iv) $\left(\frac{f}{g}\right)$ (1)

8. Let f = {(1, 1), (2, 3), (0, –1), (–1, –3)} be a linear function from Z into Z. find f(x).

Objective Assignment

1. If A = {1, 0} and B = {0, 1}, then A × B is equal to
(a) {0, 1, 1, 0} **(b)** {(1, 0), (0, 1)}
(c) {(1, 0), (1, 1), (0, 0), (0, 1)} **(d)** none of these

2. Let n(A) = 4, n(B) = 3. Then the number of relations from A to B is
(a) 12 **(b)** 7
(c) 2^{12} **(d)** 2^7

3. Let A = {a, b, c} and B = {1, 2}. Consider a relation from set A to set B. Then R is subset of
(a) A **(b)** B
(c) A × B **(d)** B × A

4. The number of functions from {a, b} into {1, 2, 3} is
(a) 6 **(b)** 5
(c) 8 **(d)** 9

5. If f(x) = $\begin{cases} 1 \text{ if } x > 0 \\ -1 \text{ if } x < 0 \\ 0 \text{ if } x = 0 \end{cases}$ then f is
(a) the absolute function **(b)** a linear function
(c) a constant function **(d)** a signum function

6. If f(x) = x^n, where n is a positive integer, then f(x) is a
(a) linear function **(b)** polynomial function
(c) absolute function **(d)** signum function

7. Which of the following functions is a polynomial function ?
(a) $\frac{x^3-1}{x+4}$, $x \neq -4$ **(b)** $x^4 + x^3 + 3x^2 - 7x\sqrt{2x^2}$
(c) $\frac{2x^2+7x+4}{3}$ **(d)** $2x^2 + 3\sqrt{x} + 4$

8. If f(x) = $\begin{cases} 2x-3, & x \geq 2 \\ x, & x < 2 \end{cases}$ then f(1) is equal to
(a) 2f(2) **(b)** f(2)
(c) –f(2) **(d)** $\frac{1}{2}$ f(2)

9. Let g(x) be a function defined on (–1, 1). If the area of the equilateral triangle with two of its vertices at (0, 0) and (x, g(x)) is $\frac{\sqrt{3}}{4}$, then
(a) g(x) = $\pm\sqrt{1-x^2}$ **(b)** g(x) = $-\sqrt{1-x^2}$ or g(x) = $\sqrt{1-x^2}$
(c) g(x) = $\sqrt{1-x^2}$ **(d)** g(x) = $-\sqrt{1-x^2}$

10. The function f(x) = 1 is the same as the function defined by
(a) f(x) = $\frac{x}{x}$ **(b)** f(x) = $\frac{x-2}{x-2}$

(c) f(x) = $\begin{cases} \frac{x}{x}, \text{If } x \neq 0 \\ 0, \text{if } x = 0 \end{cases}$ **(d)** f(x) = $\begin{cases} \frac{x}{x}, \text{If } x \neq 0 \\ 1, \text{if } x = 0 \end{cases}$

11. The domain of the function f(x) = $\frac{|x+2|}{x+2}$

(a) R – {2} **(b)** R
(c) R^+ **(d)** R – {–2}

12. The range of the function f(x) = $\frac{|x-3|}{x-3}$, $x \neq 3$

(a) {–1} **(b)** {–1, 1}
(c) {1} **(d)** {–1}

13. The range of function f(x) = |x| is
(a) (– ∞, ∞) **(b)** (0, ∞)
(c) [0, ∞] **(d)** (– ∞ , 0]

14. The range of the function f(x) = $\frac{x}{1+|x|}$ is

(a) [–1, 1] **(b)** R
(c) (–1, 1) **(d)** R – {0}

15. Which of the following relations is a function
(a) R = { (x, y) : $x^2 + y^2 \leq 9$} on R
(b) A = {1, 2, 3}, B = {1, 2, 3, 4, 5} and
R = {(x, y) : 5x + 2y is a prime number} on A.
(c) A = (1, 2, 3, 4}, B = {1, 2, 3, 4, 5, 6, 7} and
R = { (x, y)| y = $x^2 - 3x + 3$} on A.
(d) none of these

16. The domain of definition of the function

f(x) = $\sqrt{\log_{10}\left(\frac{5x-x^2}{4}\right)}$ is

(a) [1, 4] **(b)** (1, 4)
(c) (0, 5) **(d)** [0, 5]

17. The domain of the function
f(x) = $\sqrt{2-2x-x^2}$ is
(a) $-\sqrt{3} \leq x \leq \sqrt{3}$ **(b)** $-1-\sqrt{3} \leq x \leq -1+\sqrt{3}$
(c) $-2 \leq x \leq 2$ **(d)** $-2-\sqrt{3} \leq x \leq -2+\sqrt{3}$

18. The domain of definition of the real function f(x) = $\sqrt{\log_{16} x^2}$ of the real variable x is
(a) x > 0 **(b)** $|x| \geq 1$
(c) $|x| \geq 4$ **(d)** $x \leq 4$

19. The domain of the function
f(x) = $\sqrt{1-\sqrt{1-\sqrt{1-x^2}}}$ is
(a) {x : x < 1} **(b)** {x : x > –1}
(c) [0, 1] **(d)** [–1, 1]

20. If $\log_{0.3}(x-1) < \log_{0.09}(x-1)$, then x lies in the interval
(a) (2, ∞) **(b)** (1,2)
(c) (–2, –1) **(d)** none of these

Answers

Practice Assignment – I

1. (i) a = 2, b = 3 ; (ii) a = –4, b = –3.
3. (i), {(a, c), (a, d), (a, e), (b, c), (b, d), (b, e)}; (ii) {(a, d), (b, d)}
4. {(1, 3), (1, 4), (1, 5), (2, 3), (2, 4), (2, 5)}.
8. (1, 5), (2, 3), (3, 5)
9. S = {(2, 2), (2, 4), (2, 6), (2, 8), (4, 4), (4, 8), (6, 6), (8, 8)}
10. A = {x, y, z}, B = {1, 2}.
12. A = {–1, 0, 1} ; (–1, –1), (0, –1), (0, 0), (1, –1), (1, 0), (1, 1), (–1, 1)
13. A = {2, 3, 4) ; B = {3, 1} .
14. 20, 9
15. {(–1, –1, –1), (–1, –1, 1), (–1, 1, –1), (–1, 1, 1), (1, –1, –1), (1, –1, 1), (1, 1, –1), (1, 1, 1)}

Practice Assignment – II

1. (1, 1), (2, 2), (3, 3)
2. R = {(4, 6), (5, 6), (5, 8), (4, 8)}
3. Domain of R = {1, 2, 3, 4, 5, 6, 7} ; Range of R = {7, 6, 5, 4, 3, 2, 1}
4. Domain of R = {1, 2, 3, 4} ; Range of R = {8, 6, 4, 2}
5. {(1, 1), (2, 1), (3, 3), (4, 7)}
6. Domain of R = {1, 2, 3, 4, 5}, Range of R = {3, 5, 7, 9, 11}.
7. R = {(1, 1), (4, 2), (9, 3), (16, 4), (25, 5), (36, 6), (49, 7), (64, 8)}
   Domain of R = {1, 4, 9, 16, 25, 36, 49, 64}
   Range of R = {1, 2, 3, 4, 5, 6, 7, 8}
8. R = {(1, 4), (2, 8), (3, 12), (4, 16), (5, 20), (6, 24), (7, 28)}
   Domain of R = {1, 2, 3, 4, 5, 6, 7,} ;
   Range of R = {4, 8, 12, 16, 20, 24, 28}
9. (i) x 'is less than' y ;
   R = {1, 2), (1, 5), (1, 6), (2, 5), (2, 6), (3, 5), (3, 6)
   (ii) x ' is greater than' y ; R = {2, 1), (3, 1), (3, 2)}.
10. (i) Domain = {1, 2, 3, 4}, Range = {1, 4, 9, 16}
    (ii) Domain = {9, 11, 13}, Range = {4, 5, 6}
    (iii) Domain = {3, 4, 5}, Range = {–5/3, –10/8, –17/15}
11. (i) R = {(a, a) ; a $\in$ Z} $\cup$ {(a, –a) : a $\in$ Z}
    (ii) Domain = Z ; (iii) Range = Z
13. (i) No ; (ii) No ; (iii) No.
14. (i) and (iv) are relations from A to B but (ii) and (iii) are not
15. (i) A = {(4, 1), (7, 2), (10, 3), (13, 4)…….}
    Domain = {4, 7, 10, 13, 16,…}, Range = {1, 2, 3, 4,….} = N
    (ii) S = {(0, 0), (1, 1), (–1, 1), (1, –1) (–1, –1), (1, 0), (0, 1), (–1, 0), (0, –1), (2, 2), (–2, 2)
    (2, –2), (–2, –2), (2, 0), (0, 2), (–2, 0), (0, –2), (3, 0), (–3, 0), (0, –3), (0, 3), (1, 2), (2, 1), (–1, –2),
    (–2, –1), (1, –2), (2, –1), (–1, 2), (–2, 1)}

Domain = Range = {–3, –2, –1, 0, 1, 2, 3}

16. (i) Domain = {1, 2, 3, 4, 5, 6} ; Range = {1, ½, 1/3, 1/4,1/5, 1/6}
(ii) Domain = {1, 2, 3, $\sqrt{5}$ } ; Range = {1, 4, 9, 5}
(iii) Domain = {1/2, $\sqrt{2}/2$, $\sqrt{3}/2$, 1} ; Range = (π/6, π/4, π/3, π/2)

17. (i) (6, 2), (7, 3) (ii) (0, 1), (2, 3), (3, 4)
(iii) (3, 4), (–3, 4), (3, –4), (–3, –4), (0, 5)

18. {(0, 13), (0, –13), (13, 0), (–13, 0), (12, 5), (5, 12), (–12, 5), (5, –12), (12, –5), (–5, 12), (–12, –5), (–5, –12)}

19. (i) 15
(ii) {(–2, 0), (–1, 0), (0, 0), (1, 0), (2, 0), (–2, 1), (–1, 1), (0, 1), (1, 1), (2, 1), (–2, 3),I (–1, 3) , (0, 3) (1, 3), (2, 3)}.
(iii) G = {(1, 0), (2, 0), (2, 1)} ,
(iv) L = {(0, 1), (–2, 3), (–1, 3), (0, 3), (1, 3), (2, 3)}.

20. (i) {(1, 4), (2, 4), (3, 4), (4, 4)}
(ii) {(–3, 4), (–2, 3), (–1, 2), (0, 1), (1, 0), (2, 1), (3, 2)}

21. Relation = {(x, y) : x – y = 7, x, y ∈ I},
Domain = {0, 2, 4, –13,…},
Range = {–7, –5, –3, –20, ….}

22. 64

23. (i) R = {(1, 6), (2, 7), (3, 8)}
(ii) Domain = {1, 2, 3} Range = {6, 7, 8}

24. R = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (2, 2), (2, 3), (2, 4), (2, 5), (3, 3), (3, 4), (3, 5), (4, 4), (4, 5), (5, 5)}
R^{-1} = {(1, 1), (2, 1), (3, 1), (4, 1), (5, 1), (2, 2), (3, 2), (4, 2), (5, 2), (3, 3), (4, 3), (5, 3), (4, 4), (5, 4), (5, 5)}

25. (i) R^{-1} = {(2, 1), (3, 1), (3, 2), (2, 3), (6, 5)}
(ii) R^{-1} = {(3, 2), (2, 4), (1, 6)}
(iii) R^{-1} = {(8, 11), (10, 13)}

Practice Assignment – III

2. No. if A = {2, 3} and B = {1, 4}, then {(2, 1), (2, 4), (3, 4)} is a relation, but it is not a function.
3. (i) {1, 3, 5, 7}; (ii) 1 and 2.
4. (i) Function, (ii) Not a function; (iii) Not a function; (iv) Function.
6. 5, –7 and 22.
7. Not a function A = {1, 2}
8. {–9, –7, –3, 11}; –9, 1.
9. (i) 6; (ii) 4 and 25.
10. Yes; Range = {3x : x ∈ N} = {3, 6, 9, ……, 3n, ….}.
11. (i) $\left\{-1, \frac{1}{4}, \frac{3}{7}, \frac{8}{17}\right\}$; (ii) $\left\{\frac{1}{3}, \frac{3}{8}, \frac{5}{13}, \frac{7}{18}\right\}$.
12. $f = \left\{\left(-4, \frac{9}{10}\right), (-3, 1), \left(-2, \frac{5}{4}\right), (-1, 3), \left(0, -\frac{1}{2}\right), \left(1, \frac{1}{5}\right), \left(2, \frac{3}{8}\right), \left(3, \frac{5}{11}\right), \left(4, \frac{1}{2}\right)\right\}$;
Range of f = $\left\{\frac{9}{10}, 1, \frac{5}{4}, 3, -\frac{1}{2}, \frac{1}{5}, \frac{3}{8}, \frac{5}{11}, \frac{1}{2}\right\}$.
13. No. f(x) = φ(x) only when x ≠ 1.
14. 2·1
15. (i) 32 (ii) 82.4 (iii) 14 (iv) 100

16. {3, 5, 11, 13}
17. Not a function
18. Not a function

Practice Assignment – IV

1. Rational function ; $R - \left\{-\frac{7}{2}\right\}$
2. 0, 3, 0, 1, 5, –3, –3.
3. 3 and –1,
4. 0.5 and 7.5
5. (i) $(-\infty, 2)$ (ii) $[2, \infty)$
6. $R - \{1\}$
7. $[0, 3)$
8. (i) Domain = $R - \{1\}$, Range = $R - \{2\}$
   (ii) Domain = R, Range = $(-\infty, 0]$
   (iii) Domain = $R - \{-1, 1\}$, Range = $(-\infty, 0) \cup [1, \infty)$
   (iv) Domain = $[1, \infty)$, Range = $[0, \infty)$
   (v) Domain = R, Range = $[0, \infty)$
   (vi) Domain = $R - \{1\}$, Range = $\{-1, 1\}$
9. (i) $R - \{2, 6\}$ (ii) $R - \{1, 4\}$
10. (i) $R - \{-2, 2\}$ (ii) $(-2, 2)$

Practice Assignment – V

1. (i) $2(x^2 + 3x + 2)$; (ii) $4x^2 + 15x + 12$; (iii) $2(x^2 - 2)$; (iv) $6x^3 + 17x^2 + 12x$.
2. $\sqrt{x} + x$; $\sqrt{x} - x$; $x\sqrt{x}$ or $x^{3/2}$; $x^{-1/2}$ $(x \neq 0)$.
4. f(a) = a, f(b) = b, f (a + b) = a + b
7. (i) e (ii) e (iii) 0 (iv) not defined
8. f(x) = 2x – 1

Objective Assignment

1	C	11	D
2	C	12	B
3	C	13	C
4	D	14	C
5	D	15	C
6	B	16	A
7	C	17	B
8	B	18	B
9	B	19	D
10	D	20	A